

# Correlation effects in disordered conductors with spin accumulation

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We consider the effect of electron-electron interaction on the density of states of disordered paramagnetic conductor in the presence of spin accumulation and magnetic field. We show that interaction correction to electron density of states of the paramagnet may exhibit singularities at energies corresponding to the difference between chemical potentials of electrons with opposite spins. We also discuss correlation effects on conductivity in metallic as well as in hopping regimes and show that spin accumulation leads to the negative magnetoconductivity.

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## INTRODUCTION

The realization of large spin accumulation, that can be described by introducing the quasi-chemical potentials  $\mu \pm \delta\mu/2$  for spin-up and spin-down electrons in the system possessing long spin relaxation time, is important for many proposals in spintronics. In particular, the non-equilibrium spin polarization might be created by illuminating the sample with circularly polarized light [1], or can be achieved in conductors placed in contact to the ferromagnet via the spin injection mechanism [2, 3], see for a review [4, 5]. The spin injection and detection were investigated in many works and for different types of materials such as superconductors [6–8], organic polymers [9], graphene [10]. Recently there has been a progress in achieving of large spin accumulation by means of electrical spin injection in semiconductors such as GaAs, Si, Ge [11–13].

Motivated by experiments on spin accumulation, we focus on the non-equilibrium correlation effects in conductivity and density of states (DOS) in the presence of spin accumulation.

Quantum corrections to the transport and thermodynamical properties of disordered metallic conductors have been a subject of both theoretical and experimental studies. Electron-electron interaction in disordered conductors results in the singularities of electron DOS at the Fermi level and positive magnetoresistivity, for a review see [14, 15].

For example, the energy dependence of the interaction correction to the electron DOS at zero temperature has a logarithmic singularity in two dimensions  $\delta\nu(\epsilon) \propto \ln|\epsilon\tau|$ , where  $\tau$  is the electron mean free time. In addition, external magnetic field  $B$  due to Zeeman splitting leads to singularities shifted from the Fermi energy by the amount  $\pm\Omega_z$  as  $\delta\nu(\epsilon) \propto \ln|(\epsilon^2 - \Omega_z^2)\tau^2|$ , where  $\Omega_z = |g\mu_B B| \text{sign}(B)$  and  $\mu_B$  is the Bohr magneton. We define  $\Omega_z$  in such a way that it can be of either sign, depending on the direction of magnetic field. The interaction correction to the conductivity decreases in magnetic field in the limit of weak spin relaxation as [15]

$\delta\sigma \sim -\lambda\nu \ln|\Omega_z\tau|$ , where  $\lambda$  is the electron-electron interaction constant and  $\nu$  is DOS at the Fermi level per one spin. More detailed discussion can be found in [14, 15].

Correlation effects related to the spin degree of freedom can also play a significant role in magnetoresistance in the hopping conductivity regime. If the Coulomb repulsion between electrons permits the onsite double occupancy, then the probability of certain transitions under the Zeeman splitting decreases leading to positive magnetoresistivity [16–18].

In the following sections we consider correlation effects in DOS and conductivity in case of finite spin accumulation  $\delta\mu$ . We essentially rely on the calculations done for the equilibrium case, therefore we discuss only the features that distinguish the non-equilibrium case.

## DENSITY OF STATES IN METALLIC REGION

We consider disordered metallic conductor in the presence of spin accumulation and magnetic field. The spin accumulation can be obtained either by spin injection or optical orientation methods. We assume that the spin relaxation time in the system is much longer than the energy relaxation time such that we can treat the system in the energy equilibrium regime while having the non-equilibrium spin polarization. Within this assumption the interaction correction to the one particle DOS in the non-equilibrium state generated by spin accumulation can be treated in the similar way done for the equilibrium case [14]. We need to take into account the non-equilibrium exchange splitting in the definition of retarded and advanced Green functions and to modify the Fermi distribution function of electrons by introducing the chemical potential shifts of electrons with spin-up and spin-down.

Interaction correction to the one particle DOS is determined by the advanced component of Green function

$$\delta\nu_\alpha(\epsilon) = \frac{1}{\pi} \text{Im} \int \frac{d\mathbf{p}}{(2\pi)^3} \delta G_A(\mathbf{p}, \epsilon, \alpha) \quad (1)$$

where  $\alpha = \pm$  defines the spin direction and we accept

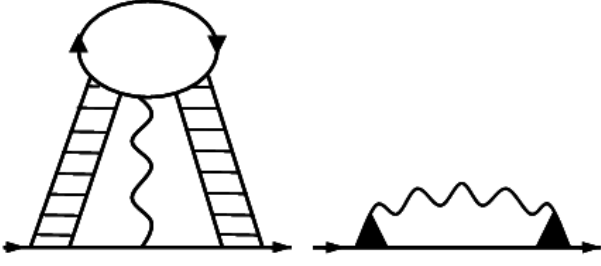


FIG. 1. Diagram for the calculation of the density of states.

the units  $\hbar \equiv 1$ . Let us consider the exchange part of the interaction correction shown in Fig. (1), where diffusion contribution to the self-energy part is given as

$$\Sigma_A(\mathbf{p}, \epsilon, \alpha) = G_R(\mathbf{p}, 0, \alpha) \frac{1}{2V} \sum_{\mathbf{q}} \int \frac{d\omega}{2\pi} \times [F(\epsilon, \alpha) - F(\omega, \alpha)] D_0^2(\omega - \epsilon, \mathbf{q}) V_A(\omega - \epsilon, \mathbf{q}) \quad (2)$$

here  $D_0(\omega, \mathbf{q}) = (D\mathbf{q}^2 - i\omega)^{-1}$  is the diffusion propagator,  $D$  is the diffusion coefficient,  $V$  is the volume of the system and screened Coulomb potential is given by the following expression

$$V_A(\omega, \mathbf{q}) = \frac{V_0(\mathbf{q})}{1 + 2\nu V_0(\mathbf{q}) \Pi_0(\omega, \mathbf{q})} \quad (3)$$

$$\delta\nu_\alpha(\epsilon) = -\frac{\nu}{V} \text{Im} \sum_{\mathbf{q}} \int \frac{d\omega}{2\pi} \left[ \frac{U_{R,\alpha}(\omega, \mathbf{q}) \tanh\left(\frac{\epsilon - \omega + \alpha\delta\mu/2}{2T}\right)}{[D\mathbf{q}^2 - i\omega - i\alpha(\Omega_z + \Omega_p)]^2} + \frac{1}{2} \frac{(U_{R,0}(\omega, \mathbf{q}) - V_R(\omega, \mathbf{q})) \tanh\left(\frac{\epsilon - \omega - \alpha\delta\mu/2}{2T}\right)}{[D\mathbf{q}^2 - i\omega]^2} \right] \quad (6)$$

where we define

$$U_{R,M}(\omega, \mathbf{q}) = \frac{\lambda}{1 - \lambda\nu \hat{\Pi}_M(\omega, \mathbf{q})} \quad (7)$$

and  $M = (0, \alpha)$ , while the polarization operator is

$$\hat{\Pi}_M(\omega, \mathbf{q}) = \frac{D\mathbf{q}^2 - iM\Omega_p/\lambda\nu}{D\mathbf{q}^2 - i\omega - iM(\Omega_z + \Omega_p)} \quad (8)$$

Here we include Zeeman splitting  $\Omega_z$  and exchange energy that is self-consistently defined as

$$\Omega_p = \frac{\lambda}{2} \int \frac{d\epsilon}{2\pi} \nu(\epsilon) [F(\epsilon, +) - F(\epsilon, -)] = \frac{\lambda\nu}{1 - \lambda\nu} [\Omega_z + \delta\mu] \quad (9)$$

We assume uniform spin accumulation and magnetic field applied parallel to the non-equilibrium magnetization.

where  $V_0(\mathbf{q})$  is the Coulomb potential and  $\Pi_0(\omega, \mathbf{q}) = D\mathbf{q}^2/(D\mathbf{q}^2 - i\omega)$ . Function  $F(\omega, \alpha)$  is determined by the Keldysh component of the non-equilibrium Green function, which after averaging over random potential becomes

$$G_K(\mathbf{p}, \omega, \alpha) = \frac{1}{i\tau} G_R(\mathbf{p}, \omega, \alpha) F(\omega, \alpha) G_A(\mathbf{p}, \omega, \alpha) \quad (4)$$

where  $G_{R/A}(\mathbf{p}, \omega, \alpha)$  are the retarded/advanced Green functions of electrons with spin direction  $\alpha = \pm$ . Function  $F(\omega, \alpha)$  in the uniform state described by the chemical potential shifts  $\delta\mu$  can be written as

$$F(\omega, \alpha) = 1 - 2n_\alpha(\omega) = \tanh\left(\frac{\omega + \alpha\delta\mu/2}{2T}\right) \quad (5)$$

We see that the integral over  $\omega$  of the first term in expression (2) proportional to  $F(\epsilon, \alpha)$  is equal to zero since all diffusion poles are in the region  $\text{Im}(\omega) < 0$ . Substituting (5) into (2) we note that expression (2) is equal to the equilibrium one under the substitution of energy  $\epsilon - \alpha\delta\mu/2$  by energy  $\epsilon$  in the expression for correction to DOS for spin direction  $\alpha$ . Therefore, the total correction to DOS for spin direction  $\alpha$  has the form

Magnetic field polarizes electron spin in the system and as a result the spin polarization  $\mathcal{S}$  in the paramagnet is a sum of two contributions coming from spin accumulation and applied magnetic field.

$$\mathcal{S} = \frac{\nu}{1 - \lambda\nu} [\Omega_z + \delta\mu] \quad (10)$$

Note, that we always assume  $\delta\mu$  to be positive. At the same time  $\Omega_z$  can be either positive or negative depending on direction of the magnetic field.

Let us now consider spin accumulation in the paramagnet in the two-dimensional limit. We take into account that  $V^{-1} \sum_{\mathbf{q}} \rightarrow \int \frac{d^2\mathbf{q}}{(2\pi)^2}$ , Coulomb interaction is  $V_0(\mathbf{q}) = e^2/q$  and we also assume small parameter  $\lambda\nu < 1$ . We find that the energy dependence of the interaction correction to the density of states in two dimensional conductor at  $|\epsilon|, \delta\mu > T$  has the form

$$\delta\nu_{\uparrow,\downarrow}(\epsilon) = -\frac{1}{4\pi^2 D} \left( \frac{1}{4} \ln \left| \frac{\epsilon \mp \delta\mu/2}{D^2 \kappa^4 \tau} \right| \ln(|\epsilon \mp \delta\mu/2| \tau) + \frac{\lambda\nu}{2} \ln(|\epsilon \mp \delta\mu/2| \tau) + \lambda\nu \ln(|\epsilon \pm (\Omega_z + \Omega_p + \delta\mu/2)| \tau) \right) \quad (11)$$

where  $\kappa = 2\pi e^2 \nu$  is the inverse static screening length.

Comparing this expression with that in the equilibrium case [14], we find that when spin polarization  $\mathcal{S}$  is zero, non-equilibrium spin accumulation splits the singularities of the density of states at  $\epsilon = 0$  [14] to  $\epsilon = \pm\delta\mu/2$  for electron with spin up and down, correspondingly. If  $\mathcal{S} \neq 0$  then the density of states exhibits singularities located at  $\epsilon = \pm|\delta\mu/2 + \Omega_z|$  for small  $\lambda\nu$  in addition to the singularities at  $\pm\delta\mu/2$ . This case is similar to the equilibrium one in the presence of magnetic field.

## CONDUCTIVITY

### Metallic region

When considering the interaction correction to conductivity in the presence of chemical potential shifts we can

use the results obtained for equilibrium [14, 15] similarly as it was done for DOS. The combined effect of external magnetic field and spin accumulation on conductivity is determined by total spin polarization. Expression for conductivity contains function  $\frac{d}{d\omega}[\omega \coth(\omega/2T)]$  that appears as a result of integration of the equilibrium Fermi distribution functions. Instead of these equilibrium functions we obtain for  $\delta\mu$ -dependent part of conductivity

$$-\frac{1}{2} \frac{d}{d\omega} \int_{-\infty}^{\infty} d\epsilon [F(\epsilon, \alpha) F(\epsilon + \omega, -\alpha) - 1] = \frac{d}{d\omega} (\omega - \alpha\delta\mu) \coth\left(\frac{\omega - \alpha\delta\mu}{2T}\right) \quad (12)$$

As a result, we obtain the expression for the spin accumulation dependent interaction correction to the conductivity

$$\delta\sigma = -i \frac{2\sigma_d}{\pi d} \frac{1}{V} \sum_{\mathbf{q}, \alpha=\pm} D\mathbf{q}^2 \int d\omega \frac{U_{R,\alpha}(\omega, \mathbf{q}) \partial_\omega \left[ (\omega - \alpha\delta\mu) \coth\left(\frac{\omega - \alpha\delta\mu}{2T}\right) \right]}{(D\mathbf{q}^2 - i\omega - i\alpha(\Omega_z + \Omega_p))^3} \quad (13)$$

Expression (13) coincides with that in the equilibrium case [15] with the corresponding spin polarization  $\mathcal{S}$ , similarly as it was done for DOS. The interaction correction to the conductivity of two-dimensional metallic system in the limit of large spin accumulation  $\delta\mu > T$  becomes

$$\sigma(\delta\mu) - \sigma(0) = -\frac{\lambda\nu e^2}{4\pi^2 \hbar} \ln |\delta\mu\tau| \quad (14)$$

while in the small spin accumulation  $\delta\mu < T$  regime the interaction correction takes the form

$$\sigma(\delta\mu) - \sigma(0) = -0.08 \frac{\lambda\nu e^2}{4\pi^2 \hbar} \left(\frac{\delta\mu}{T}\right)^2 \quad (15)$$

If the external magnetic field is applied to the system then  $\delta\mu$  in expressions (14) and (15) must be substituted with  $|\delta\mu + \Omega_z|$ . However, when considering the frequency dependent conductivity the equivalence between  $\delta\mu$  and Zeeman energy will be lost.

### Hopping region

Let us consider hopping conductivity in the presence of spin accumulation. Spin dependent contribution to the hopping conductivity arises if electron hopping involves states that permit double onsite occupancy in the presence of Coulomb repulsion [16, 17]. According to [16, 17] hopping rates are determined by correlation functions of occupation numbers of the sites involved in hopping. The

one site density matrix is given as

$$\varrho \sim \exp\left(-\frac{\epsilon_i(n_+ + n_-)}{T} + \frac{Un_+n_-}{T} + \frac{\delta\mu(n_+ - n_-)}{2T}\right) \quad (16)$$

Here  $\epsilon_i$  is the one electron energy of localized state  $i$ , counted from the Fermi level and  $U > 0$  is the intra-site electron repulsion potential.

Expression (16) coincides with equilibrium one for electrons in magnetic field. After mapping the approach developed in [16, 17] to the case of spin accumulation the hopping rate from site  $i$  to site  $k$  can be written as sum of contributions proportional to

1.  $\exp(-[\epsilon_i \pm \delta\mu/2]/T)/Z_i Z_k$  if site  $i$  has one electron and site  $k$  is empty,
2.  $\exp(-[\epsilon_i + \epsilon_k]/T)/Z_i Z_k$  if both sites are single occupied,
3.  $\exp(-[2\epsilon_i + U]/T)/Z_i Z_k$  if site  $i$  has two electrons and site  $k$  is empty,
4.  $\exp(-[2\epsilon_i + \epsilon_k + U \mp \delta\mu/2]/T)/Z_i Z_k$  if site  $i$  has two electrons and site  $k$  has one electron.

where partition function in the case of chemical potential shifts is given as

$$Z_i = 1 + 2 \cosh\left(\frac{\delta\mu}{2T}\right) e^{-\epsilon_i/T} + e^{-(2\epsilon_i + U)/T} \quad (17)$$

for  $U > 0$  assumed to be equal for all sites.

Scaling expression for the variable range hopping conductivity in this problem was obtained in [18]. For small values of spin accumulation defined by the inequality  $\frac{\delta\mu}{T}(\frac{T}{T_0})^{1/(d+1)} < 1$ , where  $T_0$  is the characteristic Mott temperature and  $d$  is the sample dimension, we obtain the expression for the spin accumulation  $\delta\mu$  dependent part of conductivity

$$\ln \left| \frac{\sigma(\delta\mu)}{\sigma(0)} \right| = -\frac{g_S g_D}{g_S + g_D} \frac{\delta\mu}{T} \quad (18)$$

where  $g_S$  and  $g_D$  are the densities of states with energies  $\epsilon_\alpha$  and  $\epsilon_\alpha + U$  near the Fermi level correspondingly. Note that in this regime  $\ln |\sigma(\delta\mu)/\sigma(0)|$  does not depend on dimension  $d$ . Again, expression (18) has to be modified in the presence of magnetic field. Spin accumulation term  $\delta\mu$  must be substituted with  $|\delta\mu + \Omega_z|$ .

## CONCLUSIONS

The singularities of DOS can be measured with a tunnelling probe electrode, where the singularities can reveal in the zero bias dips of the tunnelling conductivity [14]. We note that in spin accumulation regime both sides of the tunnelling contact will acquire chemical potential shifts. In order to resolve the splitting of singularities in DOS, the chemical potential splitting in the probe electrode has to be much smaller than that in the studied system. This can be achieved if one takes the probe electrode with much stronger spin relaxation compared to the spin relaxation in the studied system.

Let us now discuss the experimental observability of considered correlation effects. An important issue is the value of chemical potential shift  $\delta\mu$  compared to the value of the inverse spin relaxation time  $1/\tau_s$  in the system. The applicability of expressions (11), (14) and (15) requires  $\sqrt{\delta\mu\tau_s}$  to be large. We take the spin relaxation time to be  $\tau_s \sim 100$  ps and the spin accumulation  $\delta\mu \sim 1$  meV. The 1 meV spin accumulation requires correspondingly low  $< 10$  K temperatures. We obtain  $\sqrt{\delta\mu\tau_s} \sim 10$ . Also assuming the diffusion coefficient to be  $10 \text{ cm}^2 \text{ s}^{-1}$ , we estimate  $\ell_s = \sqrt{D\tau_s} \sim 300$  nm and  $\sqrt{D/|\delta\mu|} \sim 30$  nm. The above conditions on the magnitude of the spin relaxation length and spin accumulation are realizable experimentally in the ferromagnet - doped semiconductors contacts [13, 19–21]. However, the nature of this large value of spin accumulation is still under debate [22, 23].

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- [1] *Optical Orientation*, edited by F. Meier and B. P. Zakharchenya (North-Holland, Amsterdam, 1984).
  - [2] A.G. Aronov, Zh. Exp. Teor. Fiz. Pisma Red. **24**, 37 (1976); [JETP Lett. **24**, 32 (1976)].
  - [3] M. Johnson and R.H. Silsbee, Phys. Rev. Lett. **55**, 1790 (1985).
  - [4] *Spin Physics in Semiconductors*, edited by M.I. Dyakonov, (Springer, 2008).
  - [5] I. Zutic, J. Fabian, and S. Das Sarma, Rev.Mod.Phys. **76**, 323 (2004).
  - [6] P.M. Tedrow and R. Meservey, Phys. Rev. B **7**, 318 (1973).
  - [7] F.J. Jedema, B.J. van Wees, B.H. Hoving, A.T. Filip, and T.M. Klapwijk, Phys. Rev. B **60**, 16549 (1999).
  - [8] N. Poli, J.P. Morten, M. Urech, A. Brataas, D.B. Haviland, and V. Korenivski, Phys. Rev. Lett. **100**, 136601 (2008).
  - [9] W.J.M. Naber, S. Faez and W.G. van derWiel, J. Phys. D: Appl. Phys. **40**, R205 (2007).
  - [10] N. Tombros, C. Jozsa, M. Popinciuc, H.T. Jonkman and B.J. van Wees, Nature **448**, 571 (2007).
  - [11] J. Fabian, A. Matos-Abiad, C. Ertler, P. Stano and I. Zutic, Acta Phys. Slov. **57**, 565 (2007).
  - [12] D.D. Awschalom and M.E. Flatté, Nature Phys. **3**, 153 (2007).
  - [13] R. Jansen, Nature Materials **11**, 400 (2012).
  - [14] B.L. Altshuler and A.G. Aronov, *Electron-electron interactions in the disordered conductors*, Edited by A.L. Efros and M. Pollak (Elsevier, 1985).
  - [15] P.A. Lee and T.V. Ramakrishnan, Rev.Mod.Phys. **57**, 287 (1985).
  - [16] H. Kamimura, T. Takemori, and A. Kurobe, *Anderson Localization*, Edited by Y. Nagaoka and H. Fukuyama, **39**, 156 (Springer, Berlin, 1982).
  - [17] A. Kurobe and H. Kamimura, J. Phys. Soc. Jpn. **51**, 1904, (1982).
  - [18] K. A. Matveev, L. I. Glazman, P. Clarke D. Ephron and M. R. Beasley, Phys.Rev.B, **52**, 5289 (1995).
  - [19] S.P. Dash, S. Sharma, R.S. Patel, M.P. de Jong and R. Jansen, Nature **462**, 491 (2009).
  - [20] Kun-Rok Jeon, Byoung-Chul Min, Young-Hun Jo, Hun-Sung Lee, Il-Jae Shin, Chang-Yup Park, Seung-Young Park and Sung-Chul Shin, Phys. Rev. B **84**, 165315 (2011).
  - [21] S. Iba, H.Saito, A. Spiesser, S. Watanabe, R. Jansen, S. Yuasa and K. Ando, Appl. Phys. Express **5**, 023003 (2012).
  - [22] M. Tran, H. Jaffres, C. Deranlot, J.-M. George, A. Fert, A. Miard and A. Lemaitre, Phys. Rev. Lett. **102**, 036601 (2009).
  - [23] R. Jansen, A. M. Deac, H. Saito, and S. Yuasa, Phys. Rev. B **85**, 134420 (2012).